AN ISLAMIC SPACE-ENCLOSING SYSTEM

MUQARNAS

TECHNIQUES FOR DESIGNING MUQARNAS FORMS

Dan Owen / online - View online with Firefox for the best visual experience.
“It should be known that geometry enlightens the intellect and sets one's mind right. All its proofs are very clear and orderly. It is hardly possible for errors to enter into geometrical reasoning, because it is well arranged and orderly. Thus, the mind that constantly applies itself to geometry is not likely to fall into error.”


Kitab al 'Ibar, Book One, Chapter VI. The various kinds of sciences, 20. The geometrical sciences
WHAT ARE MUQARNAS?

A simple, yet seductively sophisticated, three-dimensional space-enclosing system.

» Muqarnas have a long and proud place in Islamic architecture

» In the architectural record muqarnas have been very thoroughly documented by Professor Shiro Takahashi who has provided hundreds of detailed geometric plan views of known muqarnas.

» Muqarnas can be based upon a wide variety of symmetries.

» They can create an unlimited number of forms, both new and known.

» The development of muqarnas with 3D software has allowed the creation of muqarnas forms which are not in the architectural record.

A note on transliteration: throughout I am using the Arabic transliteration, muqarnas, rather than the Persian, mogharnas.
A NOTE ON THE HISTORY OF MUQARNAS

Muqarnas were developed in Islamic countries in North Africa and Iran at about the same time in the 4th century Hijri / 10th century CE.

» They're thought to have originated with the squinch, an architectural device used to straddle square corners to allow a dome to be supported securely upon square walls.

» Professor Takahashi has found evidence of more than two thousand muqarnas architectural examples across the Islamic world.

» Among these are many famous sites, such as the Imam Mosque in Isfahan, Iran and the Alhambra in Grenada, Spain.

» Muqarnas have been primarily made of stone, polychrome tile and brick, although plaster, wooden and mirror examples exists as well. (Shiro Takahashi)
GEOMETRIC ORIGINS

The basic geometry of 3D muqarnas tiles are derived from regular polygons.

- Muqarnas may conceivably have as few as a single unique tile type.
- Realistically they need at least five. The system I've developed uses sixteen (16) unique muqarnas tiles.
- It is probably close to a practical upper limit.
- The higher the number of unique tile types the thinner the narrowest muqarnas tiles are.
- This presents engineering and materials challenges.
CREATING MUQARNAS

Choose a regular polygon (N-gon). I chose a 32-gon. Copy & rotate the N-gon N - 1 times, rotating each copy 360/N degrees around a vertex each time as shown below.

» Once complete this is the plan view of a traditional muqarnas dome.

» Each muqarnas tile is a rhombus in the plan view.

» All muqarnas tile edges are the same length in plan view and in 3 dimensions, a key property of muqarnas tiles.

» Each edge is identical, allowing all edges of all muqarnas tiles to be joined seamlessly to every edge of every other unique muqarnas tile - including itself.

» There are 16 unique rhombs and muqarnas tiles for the 32-gon.
TRADITIONAL MUQARNAS DOME

The relationship between the plan view & its associated traditional muqarnas dome.

Click image for animation.
MUGARNAS TILES

Tile 1, $11.25^\circ = 1 \times 11.25$

Tile 2, $22.5^\circ = 2 \times 11.25$

Tile 3, $33.75^\circ = 3 \times 11.25$

Tile 4, $45^\circ = 4 \times 11.25$

Tile 5, $56.25^\circ = 5 \times 11.25$

Tile 6, $67.5^\circ = 6 \times 11.25$

Tile 7, $78.75^\circ = 7 \times 11.25$

Tile 8, $90^\circ = 8 \times 11.25$

Tile 9, $101.25^\circ = 9 \times 11.25$

Tile 10, $112.5^\circ = 10 \times 11.25$

Tile 11, $123.75^\circ = 11 \times 11.25$

Tile 12, $135^\circ = 12 \times 11.25$

Tile 13, $146.25^\circ = 13 \times 11.25$

Tile 14, $157.5^\circ = 14 \times 11.25$

Tile 15, $168.75^\circ = 15 \times 11.25$

Tile 16, $180^\circ = 16 \times 11.25$

The coloring of the tiles has been an invaluable aid to recognizing patterns that make up new forms.
MUQARNAS TILES & EDGES

» The number of unique muqarnas tiles is determined by the source N-gon.
» The number of unique muqarnas tiles = \( \frac{N}{2} \) when \( N \) is even and \( \frac{N - 1}{2} \) when \( N \) is odd.
» Muqarnas tile edges are identical and must be straight lines from the plan view, but may be straight, curved, angular, or any combination from a side view.
» The edges of my tiles are 60° arcs with the top of the arc tangent to a plane parallel to the plan view.
» The edge between the upper and lower half of each muqarnas tile is a central arc whose radius increases as the apex angle increases.
» Each muqarnas tile is made of two curved surfaces joining the central arc with the upper edges and lower edges.
» Vertical 'risers' are often used in muqarnas to increase height and a sense of verticality. I chose not to use them since they do not affect the compositional possibilities. Plus, they can multiply the number of unique muqarnas tiles, thus reducing the overall design elegance. Also, from the plan view they have no area.
MUQARNAS VARIETIES

MUQARNAS TYPE VERSUS EXPANSION METHOD
MUQARNAS DOME EXPANSION

Domes are the source of muqarnas geometry and are among their primary original uses, therefore they are a logical place to start exploring more complex forms.

» One method is to make a copy of one of each of the unique muqarnas tiles
» This group is moved down and out to the periphery of the dome to begin the primary tier.
» It is copied N - 1 times at an angle of 360/N normal to the plan view around the center of the original dome.
» The required muqarnas tiles to fill the interstices are determined and copied in a similar manner.
» The secondary tier uses a duplication of the muqarnas tiles used in the primary tier with double the interstitial tiles.
» By repeating this method tertiary and quaternary tiers, etc. may be added.
Traditional domes have two unique rays. Proper placement of the rays allows the spaces to be filled with muqarnas tiles.

Click image for animation.
Traditional domes have a single unique crescent. Reversing the direction of alternate layers is pleasing.

Click image for animation.
There is an unknown but very large number of dome expansion methods. I call this one flame. It is but one more example.
VARIOUS METHODS OF DOME EXPANSION

*The ray method expands each group of similar-colored tiles separately. The other methods shown all expand as single groups. These are just a few more of what are probably hundreds of dome expansion methods. I'm more interested in exploring new families of muqarnas forms, rather than methodically and exhaustively listing members of a single family. Almost all methods (except ray) display chirality. That is, they come in right- and left-handed pairs, which is useful in design. (See crescent dome.) It may very well be that any configuration can be expanded in either a one or two color group manner. Each configuration has one of each of the fifteen different muqarnas tiles.
ONION DOME TECHNIQUE

Muqarnas onion domes use the flat, vertical 16th muqarnas tile to create a geometric condition that allows the dome to narrow below it.

» Adding the 16th tile to the traditional dome creates an 'equator', a widest radius.

» The next tile added is the 15th tile, but it and all successive tiles are rotated 180° from their normal placement. That is, all muqarnas tiles below the 'equator' are 'inside out'.

» Then the 14th tile, the 13th tile, etc. are added until one chooses to stop narrowing the 'waist'.

» If all 15 normal tiles are added below the 'equator', then the shape is closed at a point at both top and bottom. This would be a joined dome/anti-dome.

» I chose to stop at the 10th tile and then add the primary and secondary tiers. This was an arbitrary choice.

» The ray and crescent methods of dome expansion for creating primary and secondary tiers are used in the following examples.
ONION DOME RAY EXPANSION

The ray approach seems much more 3D than with the traditional dome.

Click image for animation.
The crescent expansion appears more dramatic with the onion approach.

Click image for animation.
FLATTENED DOMES

Flattened domes have a different arrangement of muqarnas tiles in the central dome.

» Several types of flattened domes have the same plan view, the same number and kind of each muqarnas tile as the traditional muqarnas dome.

» However, the tiles are in different positions than the traditional domes.

» This is possible because there are seven pairs of unique muqarnas tiles 1/15, 2/14, 3/13, 4/12, 5/11, 6/10, 7/9 that are identical in the plan view.

» Other flattened domes differ in all respects from traditional domes.

» Nonetheless, the same methods of dome expansion can be used on flattened domes.

» With less symmetry flattened domes have more types of rays and crescents, making their expansion more complex and painstaking.
This 'dome' contains the same number & type of muqarnas tiles as the traditional dome, but arranged differently. It is identical to the traditional dome in the plan view.

Click image for animation.
RAY METHOD OF FLATTENED DOME EXPANSION

This flattened dome has eight unique rays. It takes a very careful inspection to see the differences.

Click image for animation.
This flattened dome has 64 tiles in the center, 32 of which are the flat, vertical 16th tile. As a result of this it is twice the diameter of other domes.

Click image for animation.
This flattened dome also has 64 tiles in the center and is twice the diameter of other domes. Interestingly, unlike most forms the primary tier switches back to that of a non-flattened dome style.

Click image for animation.
RISING STAR DOME

This is not a true flattened dome since it has a 1:3 pitch vertical to horizontal from the lowest (outside) up to the highest (center). However, since it is similar to them rather than forms based on traditional domes, I include it with the flattened domes.

Click image for animation.
**MUQARNAS SEED**

Another method for generating muqarnas compositions uses what I call the 'seed' method. The total of the tile numbers is always equal to 32. The traditional dome has 32 #1 tiles in the center (32 × 1 = 32).

- For the top of the composition one chooses a pre-arranged or arbitrary arrangement of muqarnas tiles sharing their apex.
- The only restrictions are they do not overlap and there are no gaps.
- Then the next layer is added etc., etc. until:
  - Either the limit is reached and the next tier is begun, OR
  - The vertical muqarnas tile (#16) is added and the next tier is begun, OR
  - In reverse order the muqarnas tiles (rotated 180°) are added pulling the 'waist' in to create an 'onion' dome,
  - Which in turn allows additional tiers to be created.
This example of the 'seed' approach to muqarnas creation uses a repetition of the #1 and #3 tiles eight times for the seed. Thus, as seen at its apex the seed for this one is (1-3)8, shorthand for 1-3-1-3-1-3-1-3-1-3-1-3-1-3-1-3-1-3-1-3. In the animation it is expanded to the primary, secondary and tertiary tiers with the ray method.
This example of the 'seed' approach uses a repetition of the #2 and #4 tiles four times for the seed. So at its apex the seed for it is (2-4-2)4, shorthand for 2-4-2-2-4-2-2-4-2-2-4-2. In the animation it is expanded to the primary, secondary, tertiary and quaternary tiers with the peacock motif which appears to be unique to this form.

Click image for animation.
This example of the 'seed' approach uses a seed of 2-1-3-1-5-2-3-3-2-5-1-3-1 (no shorthand, sorry). In the animation it is expanded to the primary and secondary tiers with the ray method. It displays mirror symmetry about a single vertical axis, making it the least symmetrical form.

Click image for larger image.
Another example of the 'seed' approach. The seed for this one is 8-8-8-8, (8)4. Instead of continuing the pattern out I mirrored the secondary tier like a portal entrance. An additional mirrored copy created an internal arched space which shared an edge with the opposite side.

Click image for animation.
COMPOSITE MUQARNA FORMS

Some muqarnas forms can be combined to create composite, hybrid or mixed compositions.
Below the traditional dome the primary tier is of the ray type. The secondary tier is a complex novel form made up of two pairs of interlocking patterns.

Click image for animation.
A variety of muqarnas forms can be developed to serve as the ceilings and/or roofs of spiral staircases.

» The inner radii of the spiral staircases is determined by the muqarnas expansion from which the repeated form is derived.

» Each spiral muqarnas element needs to be shaped like a wedge or truncated wedge from the plan view.

» Of course, the leading and trailing edges of the spiral elements must join with a fixed change in elevation and angle around the center.

» The wedge angle and truncation determines the radius of the empty cylindrical center. This is similar to the expansion method for domes - but with the inner elements removed.
Crescent motif muqarnas ceiling/roof for a spiral staircase. Derived from the central dome - tightest radius.

Click image for animation.
This ray motif spiral staircase is derived from the primary tier and thus has an intermediate radius.

Click image for animation.
The crescent element used is from the secondary tier - giving it a wider radius. Stairs omitted.
ENCLOSING MUQARNAS FORMS

If muqarnas forms are placed adjacent to each other sharing a vertex or edge sometimes an enclosing form can be developed.
A dome and flattened dome share a lower, outer vertex in the same vertical plane with their central, top vertices. This form has a very complex type of symmetry. It is mirrored on the other side of the domes and is related to the whirlpool flattened dome.
This incomplete enclosing form shows how large enclosing forms can be. The 'center' of this enclosing shape is 2/5 from one side and 3/5 from the other. This pattern is so complex that the type and placement of each tile must be individually determined. It might be completed in six months, while working on other less demanding forms.

Click image for larger image.
UNIQUE MUQARNAS FORMS

When combining disparate muqarnas elements, novel forms may result. Some of these other types of muqarnas forms are difficult to categorize.
I tried unsuccessfully to create a column, but the result is interesting.
I tired of the 'super' symmetricality of muqarnas and tried unsuccessfully to create a totally irregular form. It still has two axes of mirror symmetry at right angles.

Click image for animation.
WHAT MUQARNAS AREN'T

Muqarnas are not a structural support system. They require the support of an enclosing and/or supporting architectural structure.

» As such they must be hung or otherwise attached to form a ceiling of an interior space, or

» They must be placed upon a supporting roof structure.

» Spiral staircases are a special case that imposes a variety of challenges in designing suitable muqarnas tiles, since each tile could potentially be both ceiling and roof simultaneously.
I have intentionally ignored the specific design of traditional muqarnas stone blocks for a variety of reasons.

» Historically, to 'force' muqarnas to fit onto square or rectangular walls numerous additional tile shapes were added, such as kite-shaped, half-blocks and others.

» This proliferation of tile types may have solved specific immediate problems, but it did so at the expense of realizing the elegant potential of a smaller set of 'pure' muqarnas tiles.

» My interest in muqarnas lay, not in accommodating them to known architectural forms, but in allowing them to develop inspiring new forms and methods of composing them that may be used in current architectural settings.

» The minimum muqarnas tile set, the most elegant, is also the most cost effective for manufacturing and building.

» Modern material science, structural engineering and architectural techniques should be applied to muqarnas to bring them into the present. This is a compliment, not a threat, to traditional muqarnas techniques and designs.

» For the above reasons analyses of the historically extant muqarnas are of limited interest to me and are not mentioned here.
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THE END

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Additional Information: